

Finding more NEOs with less compute using SAD tracklet rejection in HeliLinC

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ABSTRACT

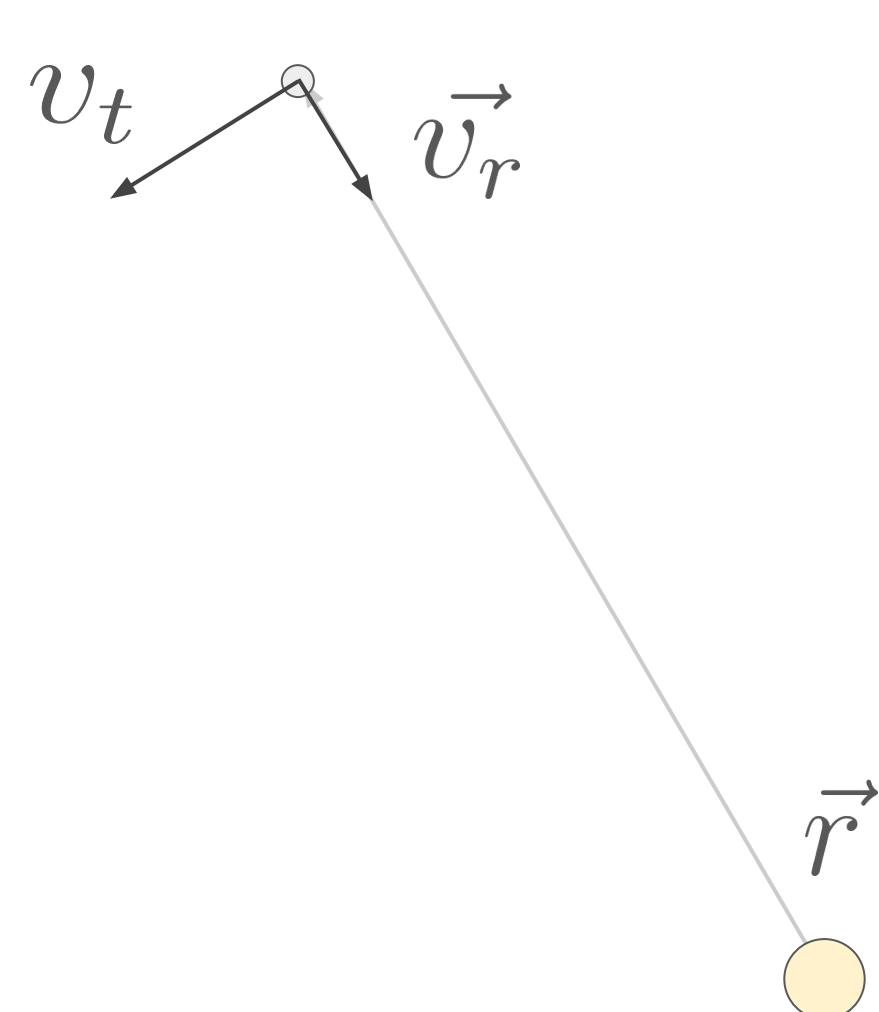
HeliLinC is an algorithm that discovers solar system objects in transient source catalogs by propagating tracklets along hypothesized orbits and clustering the resulting heliocentric positions at common reference epochs. For near-Earth objects (NEOs), which frequently have non-linearly varying heliocentric ranges as a function of time $r(t)$, the search space of hypothesized orbits is increased compared to other orbit classes. We can, however, dramatically reduce that space by testing the projected heliocentric angular sweep of a tracklet against the sweep of the hypothesized orbit. If a tracklet has a swept angle discrepancy with respect to the hypothesized orbit (a SAD tracklet), we don't have to propagate or cluster it which saves both substantial compute time and reduces the number of confounding tracklets in the clustering phase space. In the limit where we choose an object's true $r(t)$ for the hypothesis orbit and reject SAD tracklets, we can reduce propagation and clustering counts by over 95% without sacrificing object recovery rates – meaning we propagate and cluster almost nothing other than the tracklets that conform to the hypothesis orbit. In the more realistic scenario of searching for objects we do not know the orbits for, we need to increase the tolerance to account for small discrepancies from the hypothesis orbit. In these instances, we are still able to reduce the number of tracklets that need to be propagated and clustered by approximately 85% in a DP0.3 search.

THE RANGE HYPOTHESIS IMPLIES AN ORBIT ON AN UNSPECIFIED PLANE

HeliLinC^{1,2,3} works by applying range hypotheses to observed tracklets. We guess an r , $\dot{r}$ and $\ddot{r}$ at an epoch t_{epoch} and apply that propagated time varying $r(t)$ to all the tracklets we're searching in order to define a 3D heliocentric position for them all. Consider the heliocentric position and decomposed radial and tangential velocity vectors with an unknown $\hat{v}_t$ for an object below:

Given an observation $\hat{r}$ at t_{epoch} , assert r , $\dot{r}$, and $\ddot{r}$ then:

$$\ddot{r} = \frac{-GM}{r^2} + \frac{v_t^2}{r}$$

$$v_t = \sqrt{r(\ddot{r} + \frac{GM}{r^2})}$$


Now select a random plane $\hat{n}$ to propagate v_t in:

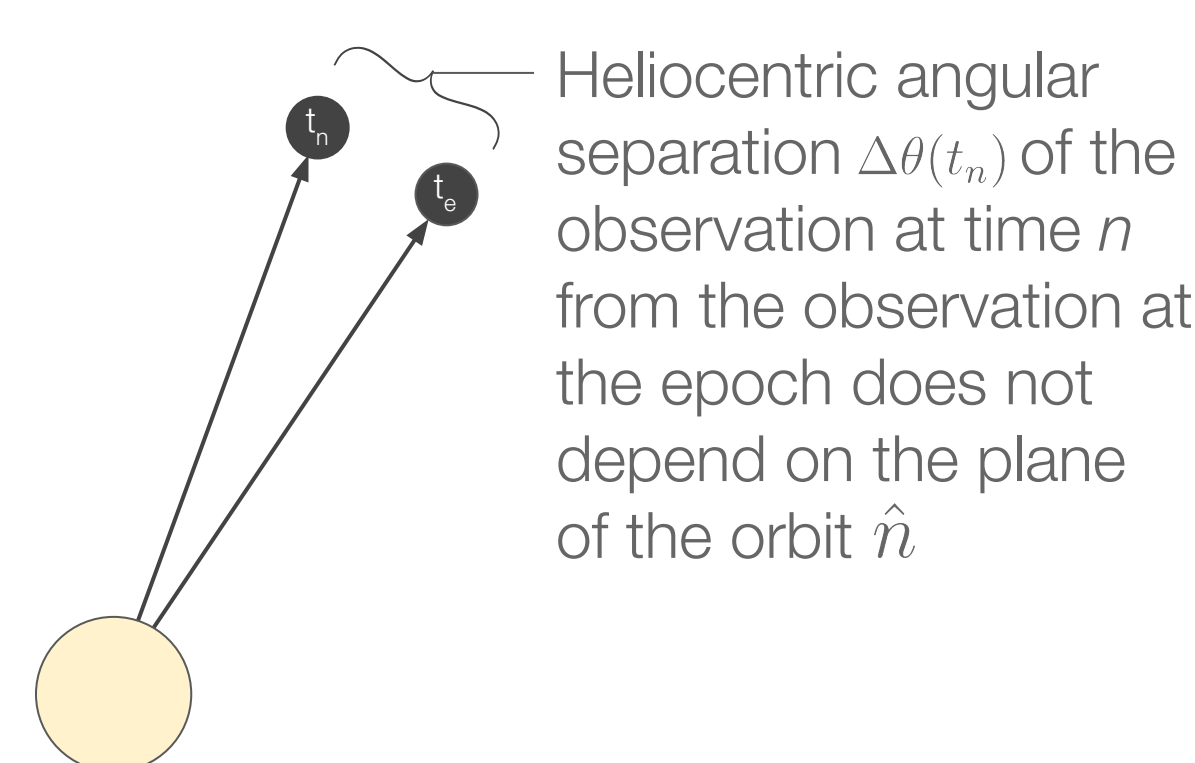
$$\hat{v}_t = \frac{\hat{r} \times \hat{n}}{|\hat{r} \times \hat{n}|} \quad \text{where} \quad \hat{n} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Then $\vec{v} = v_r \hat{r} + v_t \hat{v}_t$

Given $\vec{r} \equiv r \cdot \hat{r}$ and $v_r \equiv \dot{r}$ at t_{epoch}

We can construct a state vector: $\vec{r}, \vec{v}, t_{epoch}$ from our range hypothesis and $\hat{n}$

By specifying r , $\dot{r}$ and $\ddot{r}$ at an epoch t_{epoch} and then choosing an arbitrary plane $\hat{n}$ (we use $\hat{z}$: the x-y plane in this instance) to propagate in, we can calculate $r(t)$ for the hypothesis from the state vector we obtain. If you choose any other $\hat{n}$, the $r(t)$ you calculate will be the same as well. $r(t)$ is thus independent of the plane $\hat{n}$. Furthermore, if you calculate the angular separation from the epoch as a function of time $\Delta\theta(t)$ in the heliocentric projection along any plane, that amount of separation will also be the same for any $\hat{n}$. The process to verify this is below:

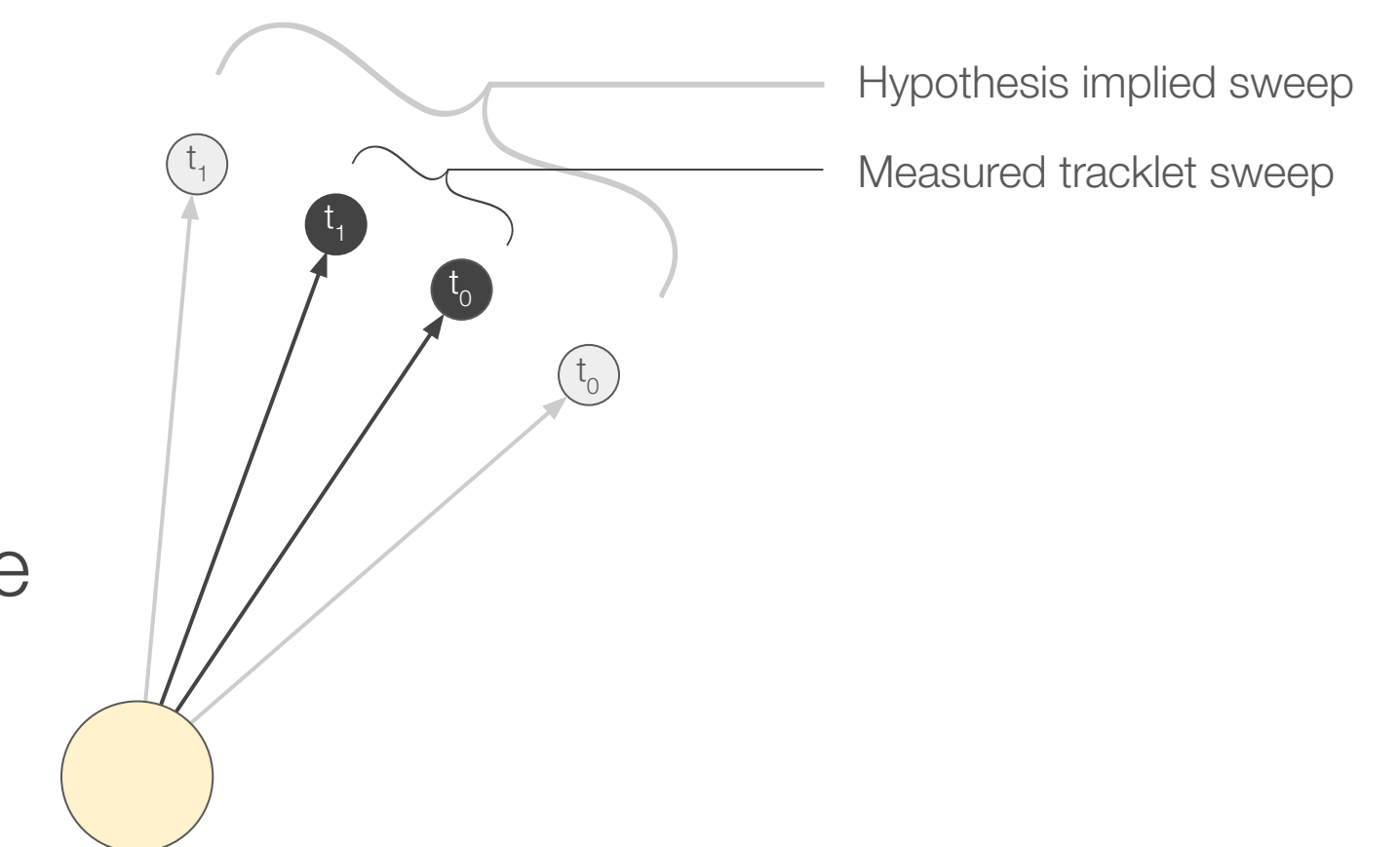


1. Select a random heliocentric unit vector $\hat{r}$ (e.g. [1,0,0])
2. Apply a range hypothesis to this vector (e.g. $r=3$, $\dot{r}=0$, $\ddot{r}=-1e-5$) to get $\vec{r}$ at the hypothesis epoch (let's say $t_{epoch}=0$)
3. Calculate v_t at the hypothesis epoch from the equation above
4. Cross $\hat{r}$ with a random vector representing a plane $\hat{n}$ (e.g. [0,0,1]) and normalize to get $\hat{v}_t$: a direction perpendicular to $\hat{r}$ to apply v_t in
5. Rotate $\hat{v}_t$ through 360° in 1° increments about the axis pointing towards $\hat{r}$ and keep each of the unit vectors representing 360 $\hat{v}_t$ values
6. Create the velocity vectors at the hypothesis epoch with the equation $\vec{v} = v_r \hat{r} + v_t \hat{v}_t$ for each $\hat{v}_t$
7. Propagate $\vec{r}$ and each $\vec{v}$ to n different times (e.g. $t=1,2,3$) for all 360 velocity vectors
8. Calculate $r(t)$ and $\Delta\theta(t)$ for all n times and 360 velocity vectors: $r(t)$ and $\Delta\theta(t)$ should be the same for all pointings of $\hat{v}_t$

EXPLOITING ANGULAR MOTION KNOWLEDGE

HeliLinC's range hypothesis forces all tracklets to move with the same in-plane motion along the hypothesized orbit with the plane determined by the individual tracklet's heliocentric positions at the hypothesized range. The range hypothesis thus constrains everything but the plane of the orbit.

As a result, we can test the angular sweep $\Delta\theta t_f - \Delta\theta t_i$ of a tracklet in the heliocentric projection against the expected sweep for the hypothesis orbit from the first to last observation times of a tracklet and check for deviation. If there is a substantial discrepancy, the tracklet cannot represent the hypothesized orbit because its motion along the orbit is different from the hypothesis orbit's motion. Therefore, we can entirely skip propagation and clustering for



tracklets with a **swept angle discrepancy** to the hypothesis orbit. We refer to these rejected tracklets as SAD tracklets. This optimization reduces both the compute requirements and the confounding tracklets in the clustering phase space.

HELIOLINC RESULTS WITH RUBIN DP0.3 DATA

We use Rubin DP0.3⁴ data (preview data that simulates observations in the solar system for Rubin's LSST) to search a two week window for NEOs using the author's implementation of HeliLinC. Using perfect hypotheses for a NEO-only subset of DP0.3 without SAD tracklet rejection, we must propagate and cluster 410,320 tracklets from 258 exact hypotheses to recover all 258 NEOs in the data. When we impose a SAD tracklet constraint limiting the swept angle discrepancy to 1%, we reduce that propagation and clustering count to 11,993 – a 97.1% reduction with no loss in object recovery.

For the more realistic scenario of searching for NEOs amongst all DP0.3 transients without a priori orbit information, we would have to propagate and cluster 2,454,259,912 tracklets without SAD tracklet rejection from a grid of 1,352 hypotheses to achieve 95% object recovery. Using a SAD tracklet rejection tolerance of 5% to account for our hypotheses not being perfect anymore, the number of tracklets propagated and clustered is reduced by 86.7% to 327,444,550 with no loss in object recovery. The tables below summarize the results:

NEO only transient search with perfect hypotheses		
SAD Tracklet Tolerance	Propagated Tracklets	Objects Recovered
None	410,320	100%
5%	54,098	100%
4%	43,572	100%
3%	33,059	100%
2%	22,561	100%
1%	11,993	100%

Full transient search of 1,352 gridded hypotheses		
SAD Tracklet Tolerance	Propagated Tracklets	Objects Recovered
None	2,454,259,912	95%*
5%	327,444,550	95%

*This number represents the best case for the hypotheses grid. 2.5B tracklets cannot be tested with author's computing resources.

CONCLUSION

SAD tracklet rejection in a HeliLinC search takes advantage of our knowledge of the expected in-plane motion of a hypothesis orbit to discard tracklets that do not have the same motion along their orbit as the hypothesis we're testing *before* we take the computationally expensive steps of propagation and clustering. This increases search speed and decreases the number of confounding tracklets in phase space without a loss of object recovery for the hypothesis orbit.

REFERENCES

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2. "Solar System Processing using HeliLinC2" - Heinze, A., Eggl, S et al., 2022, <https://github.com/lsst-dm/heliolinc2>
3. "HeliLinC: A Variation in 6 Dimensions" - Engebret, B., 2022, <https://benengebret.org/heliolinc/>
4. The DP0.3 data set was generated by members of the Rubin Solar System Pipelines and Commissioning teams, with help from the LSST Solar System Science Collaboration, in particular: Pedro Bernardinelli, Jake Kurlander, Joachim Moeyens, Samuel Cornwall, Ari Heinze, Steph Merritt, Lynne Jones, Siegfried Eggl, Meg Schwamb, Grigori Fedorets, and Mario Juric.

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